Activity 36   The general quadratic formula

Aim: Solve quadratic equations by completing the square.
       Derive the general quadratic formula.
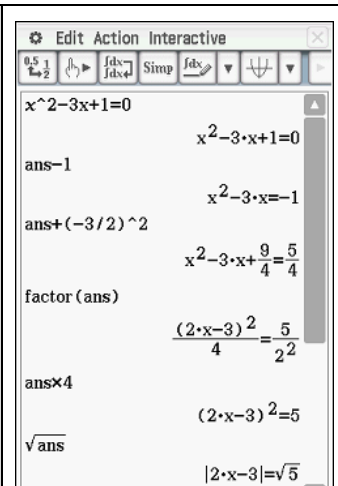
Carry out the following algebraic steps on ClassPad to solve the equation
$x^2 - 3x + 1 = 0$ as shown.

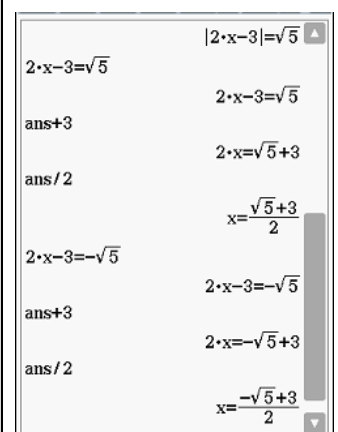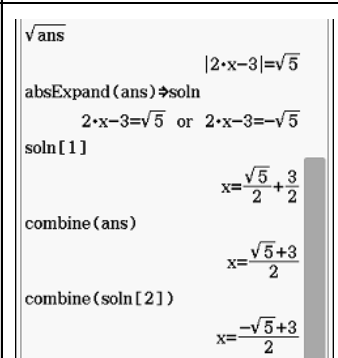|	
Enter equation and complete the square • Enter the equation and press $\boxed{\text{EXE}}$ • Press $\boxed{-}$ and $\boxed{1}$ to subtract 1 from both sides • Add $\left(\dfrac{-3}{2}\right)^2$ to both sides Half of the coefficient of x squared • Factor the result • Multiply both sides by 4, to simplify **Square root both sides** • Square root both sides	Edit Action Interactive x^2−3x+1=0 $x^2-3\cdot x+1=0$ ans−1 $x^2-3\cdot x=-1$ ans+(−3/2)^2 $x^2-3\cdot x+\frac{9}{4}=\frac{5}{4}$ factor(ans) $\frac{(2\cdot x-3)^2}{4}=\frac{5}{2^2}$ ans×4 $(2\cdot x-3)^2=5$ $\sqrt{\text{ans}}$ $\lvert 2\cdot x-3\rvert=\sqrt{5}$
Complete the solution • Highlight $2x-3$ in the answer and drag into the next line, press $\boxed{=}$ • Highlight $\sqrt{5}$ and drag into next line to complete an equation describing one solution • Complete the manual solution for the equation. • Repeat for the other equation $2x-3=-\sqrt{5}$	$\lvert 2\cdot x-3\rvert=\sqrt{5}$ $2\cdot x-3=\sqrt{5}$ $2\cdot x-3=\sqrt{5}$ ans+3 $2\cdot x=\sqrt{5}+3$ ans/2 $x=\frac{\sqrt{5}+3}{2}$ $2\cdot x-3=-\sqrt{5}$ $2\cdot x-3=-\sqrt{5}$ ans+3 $2\cdot x=-\sqrt{5}+3$ ans/2 $x=\frac{-\sqrt{5}+3}{2}$
OR **This will work with numerical coefficients** • Select [Action │Equation/Inequality │absExpand] tap $\boxed{\text{ans}}$ and tap $\Rightarrow$soln to store both equations • Enter soln[1] • Select [Action │ Transformation │ Combine] • For the second solution in one step enter combine(soln[2])	$\sqrt{\text{ans}}$ $\lvert 2\cdot x-3\rvert=\sqrt{5}$ absExpand(ans)⇒soln $2\cdot x-3=\sqrt{5}$ or $2\cdot x-3=-\sqrt{5}$ soln[1] $x=\frac{\sqrt{5}}{2}+\frac{3}{2}$ combine(ans) $x=\frac{\sqrt{5}+3}{2}$ combine(soln[2]) $x=\frac{-\sqrt{5}+3}{2}$

1. Annotate this solution by explaining the steps in lines 3,4 and 6.

$$x^2 - 3x + 1 = 0$$

$$x^2 - 3x = -1$$

$$x^2 - 3x + \frac{9}{4} = \frac{5}{4}$$ _____

$$\frac{(2x-3)^2}{4} = \frac{5}{4}$$ _____

$$(2x-3)^2 = 5$$

$$2x - 3 = \pm\sqrt{5}$$ _____

$$x = \frac{3 \pm \sqrt{5}}{2}$$

2. Edit your ClassPad solution to help write solutions to the following equations using the method of completing the square.

a) $x^2 - 3x - 8 = 0$

b) $z^2 + 5z - 8 = 0$

3. Edit your ClassPad solution to help write solutions to the following equations using the method of completing the square.

    a) $x^2 + 5x + c = 0$

    b) $x^2 + bx + c = 0$

4. The general quadratic equation $ax^2 + bx + c = 0$ has solution

    $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

    Prove this by completing the square and solving $x^2 + \dfrac{b}{a}x + \dfrac{c}{a} = 0$.

Learning notes

This activity aims to derive the solution to the general quadratic equation. Many students have happily used the solution without worrying about the derivation.

Q2   Do this by editing your work. Scroll back in Main to edit the equation and make the required changes thereafter.

There are some commands that may be new.

- absExpand
- combine – write as a single fraction or put over a common denominator